## Frittenden Church of England Primary School

## Maths Calculation Policy

| Written by: S Hancock | February 2021 |
| :--- | :--- |
| Agreed by governors: |  |
| To be reviewed: | March 2024 |
| Signed by Chair of Governors: |  |
| Signed by Headteacher: |  |

## Intent

## Introduction

The mathematics curriculum enables children to learn and build upon prior learning in a multitude of ways. Individual lessons use strategies that stimulate and inspire children; mathematical games, stories with an arithmetical focus; activities that involve movement (active learning) and references to realworld scenarios are some of the techniques that are used at Frittenden Primary School to provide children a motivating and engaging curriculum. As each lesson builds upon a previous step, learning points are constantly being referred to and used in subsequent sessions. This provides the opportunity for revision and applying skills continuously throughout the year.

Children enjoy maths -
'I love maths because I can check my work myself using the inverse.'
(Year 3)
I have loved maths lessons outdoors. We learnt about position of shapes after reflection or translation.' (Year 5)

Pupils are able to demonstrate their perseverance and courage within the subject by approaching challenging problems with confidence and without fear of failure. Children should be taught to reference the word FAIL as First Attempt In Learning; children are encouraged to build upon mistakes; find, challenge and question errors which in turn leads to new learning opportunities. Children are never rebuked for mistakes; every effort is celebrated and treated as a learning opportunity. Teachers may even make their own 'mistakes', this provides an opportunity for children to witness first hand appropriate responses for dealing with such situations - even asking for assistance from the children in this example would be encouraged.

Our Christian values underpin all aspects of our teaching and learning. The high aspirations and expectations the teaching team have for the children ensure that no child is given a learning limit or ceiling and that every child is given the opportunity to 'shine'. As mentioned earlier, every effort and opportunity for praise should be celebrated. The school values are integrated into every maths lesson as follows:

Nurturing Perseverance - An increasing level of challenge is evident and appropriate for each child. Children approach mathematical work with confidence and a positive attitude.

Ambition - Children (and their teachers) want to succeed. 'Challenges' are attempted and never feared. There is an open dialogue between teacher and child evident within the classroomand also within exercise books. All children, regardless of their starting point or prior attainment are provided with opportunities that enable them to develop their mathematical thinking.

Responsible and Respectful - Responses from other children are encouraged and never ignored. All offers ofinput are celebrated and encouraged through talk partners.

Kindness - Children show respect to teachers and their peers. They work in groups to tackle problems, support each other and apply knowledge.

- build a positive attitude and approach to mathematics; seen as an interesting and attractive subject inwhich all children gain success.
- Build upon prior knowledge and have access to a high-quality maths curriculum that is both challengingand enjoyable.
- Ensure children are provided with a variety of mathematical opportunities, which will enable them tomake the connections.
- ensure children are confident mathematicians who are not afraid to take risks and learn from mistakes.
- develop an ability to express themselves fluently, to talk about the subject with assurance, usingcorrect mathematical language and vocabulary.
- develop mathematical skills and knowledge and recall of basic facts and the four operations
- make use of diagrams, models and informal notes to help record steps and part answers.


## Implementation

## Planning

Maths is a core subject in the National Curriculum. Teachers have access to a variety of websites and planningto support their planning process which enables them to find high quality Maths resources. Planning involves the use of the White Rose small steps to ensure learning in cumulative and progressive. A variety of visual andpractical approaches support children with their mathematical fluency, vocabulary and the development of their confidence to reason and solve problems.

Planning is undertaken at three levels:

## 1. Long term: National Curriculum

The National Curriculum 2014 has 3 central aims:

- Become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately. In order to achieve this, we need to provide opportunities for children to investigate numbers by counting, cardinality (how many there arein the group), comparison and composition. They need to practice decomposing and recomposing numbers, recalling number bonds and multiplication tables to improve mathematical fluency.
- Reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language. The conversations wehave and questions we ask are key to developing reasoning skills. We can ask children to describe, explain, convince others, justify and prove to promote their reasoning skills. Adults can support children to develop reasoning by modelling, using mathematical language (also displayed in classrooms), proving or disproving statements (developing reasoning skills), group work including cooperative learning and making personal notes and recordings.
- Can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions. Activities should be provided where children can solve number problems, practical problems and missing number problems.

Problem solving is not just about solving the problem, it is about how they solved the problem. What strategies and mathematical concepts did they use? All pupils should have the opportunity to apply their mathematics to solve problems. The useof mathematical language, modelling and the bar model can all help support children to develop their problem-solving skills. Higher attaining children need to solve problems that require more demanding reasoning and problem-solving skills rather than harder numbers.

## 2. Medium term

The sequencing of teaching reflects the needs of the learners. Place value and arithmetic are given priority atthe start of each academic year as these are the building blocks for mathematical learning. The small steps ensure that previous learning is built upon, this -along with ongoing formative assessment and analysis - enables the teacher to personalise learning and resources in response to the children's ability, starting pointand individual next steps.

## 3. Short term planning

Short term planning is carried out weekly by the class teacher supported by the use of the White Rose Premiumresources, Times Tables Rockstars, Mathletics and our Calculation Policy.

- Daily lessons include a clear lesson intention 'LT I can' and clear success criteria.
- Daily lessons are taught using a variety of approaches covering some (or all) of the CPA approach ineach lesson.
- Lessons begin with an arithmetic starter (using flashback 4 or last term, week, lesson, mistake) to buildupon arithmetical methods and encourages children to make links between learning.
- Teaching pedagogy is constructed to meet the needs of all learners and enhance knowledge retentionsuch as retrieval practice, group work, partner work, mathematical learning through stories and creating links in context.


## Concrete, pictorial and Abstract (CPA) approach

At Frittenden Primary School, we recognise that the Concrete Pictorial Abstract (CPA) approach is highly effective in the teaching of Maths to develop conceptual understanding. This approach will vary between yeargroups and the individual abilities of children within each class.

Objects, pictures, words, numbers and symbols are everywhere. The mastery approach incorporates all of these to help children explore and demonstrate mathematical ideas, enrich their learning experience and deepen understanding. Together, these elements help cement knowledge so pupils truly understand what
they've learnt. All pupils, when introduced to a key new concept, should have the opportunity to build fluency in this topic by taking this approach. Pupils are encouraged to physically represent mathematical concepts.
Objects and pictures are used to demonstrate and visualise abstract ideas, alongside numbers and symbols.

## Concrete - The doing stage

There is a clear focus on the use of manipulatives and visual images to support understanding in every year group. Each new concept or calculation strategy will be introduced using appropriate manipulatives, giving thechildren a clear picture of the theoretical mathematics, they are learning. It is important that children have access to a wide range of manipulatives in every year group and, consequently, we encourage children to be independent in their use of manipulatives throughout the school and access resources as they see fit. This is the foundation for conceptual understanding.

Concrete resources that may be found in classrooms will include:


These resources will vary depending on year group and individual needs.

## Pictorial - The seeing stage

A child has sufficiently understood the hands-on experiences performed and can now relate them torepresentations, such as a diagram or a picture of the problem.

## Abstract- The symbolic stage

A child is now capable of representing problems by using mathematical notation, for example $10 \div 2=5$

## Assessment

1. Summative/reported
2. Standardisation (Year 1-6) - Termly assessments for each year group (White Rose and Past SATspapers - Year 6)
3. Diagnostic - Pupil and cohort gap analysis. White Rose Block Pre Tests \& End of Block Tests
4. Formative - See Feedback Policy for daily formative assessment opportunities

Assessment for learning approaches at different stages of the lesson.

## Impact

Pupils will leave us prepared for the next stage in their lives with:

- Quick recall of facts and procedures
- The flexibility and fluidity to move between different contexts and representations of mathematics
- The ability to recognise relationships and make connections in mathematics
- Confidence and belief that they can achieve
- The knowledge that maths underpins most of our daily lives
- Skills and concepts that have been mastered
- Have a positive and inquisitive attitude to mathematics as an interesting and attractive subject in whichall children gain success and pleasure.

A mathematical concept or skill has been mastered when a child can show it in multiple ways, using the mathematical language to explain their ideas, and can independently apply the concept to new problems inunfamiliar situations and this is the goal for our children.

## Models and Images

The following models and images are used to support the teaching and learning of different concepts. They may be used by the teacher, child or more commonly by both to aid understanding of the various topics.

| Part-Whole Model | Bar Model | Cubes |
| :---: | :---: | :---: |
|  |  | $\begin{aligned} & 7=4+3 \\ & 7=3+4 \\ & 7-3=4 \end{aligned}$ |
| Ten Frames | Bead Strings | Number lines and Tracks |
| $4+3=7$ <br> 4 is a part. <br> $3+4=7 \quad 3$ is a part. <br>  <br> $7-4=3$ <br> Then <br> Now $4+3=7$ | -00-00000000--000-9000000- $\begin{aligned} & \text {-00000-00000-00000- } \\ & \begin{array}{l} 5 \times 3=15 \quad 15 \div 5=3 \\ 3 \times 5=15 \end{array} \end{aligned}$ | $5+3=8$ $35+37=72$ |
| Place Value Charts | Base 10 / Dienes | Number Shapes |
|  | III! $11!{ }^{68+2=34}$ $72 \div 3=24$ |  |

At Frittenden, we recognise the importance of ensuring children have a strong foundation of mathematical knowledge and this is developed from sesisons within the foundation stage. Modelling, repetition and exposure to a variety of scenarios involving number are central to introducing this strand. By the end of the year, children should be able to: verbally count to numbers beyond 20, compare quantities, sort and begin to recognise odds, evens and doubles, subitise and build automatic recall ofnumber bonds to 5 (some children to 10 ), include and recognise the use of ' 0 ' in language and representations.

The following five principles are key to children gaining a deep understanding of number.

\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \& \multicolumn{2}{|l|}{\begin{tabular}{l}
The one-one principle. \\
This involves children assigningone number name to each object being counted.
\end{tabular}} \& The stable-order principle. Children understand that when counting, the numbers have to be in a certain order. \& \begin{tabular}{l}
The cardinal principle. \\
Children understand that the number name assigned to a group of objects is the total amount of objects within that group.
\end{tabular} \& The abstraction principle. Children can understand that anything can be counted, including things that cannot be touched (sounds, movements etc) \& The order irrelevance principle. This involves children understanding that the order we count a group of objects is irrelevant. The total will be the same. \\
\hline \& \multicolumn{3}{|l|}{Concrete} \& \multicolumn{2}{|c|}{Pictorial} \& Abstract \\
\hline a
g
e

2 \& \multicolumn{5}{|l|}{} \& | Although children are not expected to write numerals, these will be modelled, discussed and encouraged to form the digits $0-9$. |
| :--- |
| Informal jottings could be used to record their thinking. | <br>

\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{4}{|c|}{ADDITION－KS1（Years 1\＆2）} \\
\hline \& Concrete \& Pictorial \& Abstract \\
\hline \begin{tabular}{l} 
S \\
t \\
a \\
g \\
e \\
\\
\hline
\end{tabular} \& Use part part whole model，cubes and bead strings to addtwo numbers together as a group or in a bar． \& Use jottings to represent numbers． \& \begin{tabular}{l}
Children will record their calculation using a pictorial method along with a calculation using numbers and symbols．
\[
11+4=15
\] \\
They may use their fingers to support their mental methods
\(\qquad\)
\end{tabular} \\
\hline S
t
a
g
e

2 \& \begin{tabular}{l}
Grouping objects to add Children will use dienes cubes to add larger numbers where regrouping is not required． <br>
They will also use a bead string to add larger numbers bycounting in tens and ones

 \& 

Number line <br>
Start at the larger number on the number line and count on in ones or in one jump to find the answer．Children will show their representations from the concrete method using pictures．

$$
\begin{gathered}
14+12=26 \\
\prod_{\text {吅品 }^{\prime}+\left\|_{\text {吅 }}=\right\| \|_{\text {品品 }}} .
\end{gathered}
$$ <br>

Numbers will get progressively larger throughout the key stage．Children will be able to add tens and ones using an empty number line．

 \& 

Children will record their calculation using a

$$
27+10=37
$$ pictorial method along with a calculation using numbers and symbols． <br>

Children will begin to add multiples of tens．
\end{tabular} <br>

\hline S
t
a
g

e \& \begin{tabular}{l}
Partitioning <br>
Children will add larger numbers where they will need tojoin， regroup and count． <br>
Children will also use bead strings to add numbers together using groups of tens andones to count on．

 \& 

Number line <br>
Use an empty number line to count in tens and then ones． <br>
When confident：

 \& 

Partitioning <br>
Children will begin to use the partitioning method． <br>
Tens and ones will be added to form

$$
20+40=60
$$ partial calculations and then these will be added together to find the total．

\end{tabular} <br>

\hline
\end{tabular}

## ADDITION - Lower KS2 (Years 3 \& 4)

\begin{tabular}{|c|c|c|c|c|}
\hline \& Concrete \& Pictorial \& \& Abstract \\
\hline \begin{tabular}{l} 
S \\
t \\
a \\
g \\
e \\
\\
\hline
\end{tabular} \& Use dienes cubes to consolidate learning from KS1. Ensure children are confident at using these to join, regroup and count. This will support them moving onto the next stage of column addition. \& \multicolumn{2}{|l|}{\begin{tabular}{l}
Number line \\
Consolidate their learning from KS1 by using an empty number line to count larger numbers.
\end{tabular}} \& \begin{tabular}{l}
Partitioning \\
Children will consolidate using the partitioning method. The layout will begin to form a written method to support further progress onto the column method. Hundreds, Tens and ones will be added to form partial calculations and then these will be added together to find the total.
\end{tabular} \\
\hline S \&  \& Children can draw a representation of the grid to further support their understanding, carrying the ten underneath the line. \& \begin{tabular}{|c|c|}
\hline 00 \& 0 \\
\hline 0 \& 0 \\
\hline 5 \& 1 \\
\hline 6 \& \\
\hline
\end{tabular} \& Expanded column method - Formal method Children to use the Expanded Column Method. Start by partitioning the numbers before the formal column to show the exchange. Once confident, they can move onto the column method in stage 3.
\[
\begin{array}{r}
176 \\
+\begin{array}{r}
147 \\
13 \\
110 \\
+
\end{array}(70+6) \\
\underline{200} \\
\hline \underline{323} \\
\hline
\end{array}
\] \\
\hline S
t
a
g
e

3 \& Children will add larger numbers where they will need to exchange place value counters or dienes cubes. \& Children can draw a representation of numbers. \& the grid using larger \& \begin{tabular}{l}

| Column method - |
| :--- |
| Formal method <br> Column Method <br> for addition to be <br> used. |$+4$ <br>

\hline
\end{tabular} <br>

\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{4}{|c|}{ADDITION - Upper KS2 (Years 5 \& 6)} <br>
\hline \& Concrete \& Pictorial \& Abstract <br>
\hline S
t
a
g
e

1 \& \begin{tabular}{ll|l|l|l|l|l}

| Introduce decimal |
| :--- |
| value counters |
| model regrouping |
| addition. | \& tens \& ones \& tenths \& hundredths \& | place |
| :--- |
| and | <br>

for

 \& 

Children will draw their representations showing where they have regrouped. <br>
6
\end{tabular} \&  <br>

\hline g
e
2 \& Please note: Concrete apparatus and pictorial representations should still be used to support children who may be struggling with abstract concepts. Concrete apparatus should be readily available for lower achieving children and these with SEND. \& Children will begin to use the bar model when problem solving. Jottings and calculations should be recorded to show their processes. \&  <br>
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|}
\hline \& \multicolumn{3}{|c|}{SUBTRACTION - KS1 (Years 1\&2)} <br>
\hline \& Concrete \& Pictorial \& Abstract <br>
\hline S
t
a
g
e

1 \& \begin{tabular}{l}
Taking objects away <br>
Use part whole model, cubes and bead strings to subtract two numbers together by moving objects away from the group.

 \& Use jottings to represent numbers. Children will learn to cross out drawn objects to show what has been taken away. \& 

Children will record their calculation using a pictorial method along with a calculation using numbers and symbols.

$$
11-4=7
$$ <br>

They may use their fingers to support their mental methods
\end{tabular} <br>

\hline S
t
a
g
e

2 \& \begin{tabular}{l}
Children will use dienes cubes to larger numbers where exchanging required. Children will lay out the number using the dienes cubes move the second number away to subtraction. <br>
They will also use a bead string to numbers by counting in tens and

 \& 

Number line <br>
Children will begin to draw their own number lines. Start ${ }^{\circ}$ at the larger number on the number line and count back in ones or in one jump to find the answer. <br>
Numbers will get progressively larger throughout the key stage. Children will be able to subtract tens and ones using an empty number line. <br>
Children will show their representations from the concrete method using

$$
43-21=22
$$ pictures.

 \& 

Children will record their calculation using a pictorial method along with a calculation using numbers and symbols.

$$
25-12=13
$$ <br>

Children will begin to subtract multiples of tens.

$$
\begin{aligned}
& 25-10 \\
& 25-10=15
\end{aligned}
$$

\end{tabular} <br>

\hline S
t
a
g
e

3 \& Children will begin to use place value counters and dienes cubes to show how to exchange between units of number. They will be able to change 1 ten and exchange it for 10 ones. \& \begin{tabular}{l}
Empty number line -Use an empty number line to count back in tens and then ones. <br>
When confident:

 \& 

Partitioning method <br>
Children will begin to use the partitioning method to form calculations.

$$
\begin{aligned}
& 47-23=24 \\
& 47-20=27
\end{aligned}
$$

\end{tabular} <br>

\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{4}{|c|}{SUBTRACTION - Lower KS2 (Years 3 \& 4)} <br>
\hline \& Concrete \& Pictorial \& Abstract <br>
\hline S
t
a
g
e

1 \& \begin{tabular}{l}
Children consolidate and useplace value counters and dienes cubes to show how toexchange between units of number. They will be able to change 1 ten and exchange it for 10 ones. <br>
They will be able to begin to lay this out like the column method and removing counters or cubes away to representtaking away.

 \& 

Consolidate their learning from KS1 by using an empty number line to calculate larger numbers. <br>
Children will also be able to draw representations of dienes cubes and place value counters by crossing out the number being taken away. <br>
Develop the use of empty number line with calculations that bridge 100: <br>
Count on to find small differences:

 \& 

$$
\begin{array}{r}
908 \\
-305-24=23 \\
-305-\frac{40+7}{20+4} \\
\hline 603 \\
\hline
\end{array}
$$ <br>

Children to further secure their knowledge using the partitioning method but will start to lay their work out using the column method approach.
\end{tabular} <br>

\hline \& Children begin to set out HTU HTU using dienes cubes and place value counters (that cross the tens boundary) in columns and record as column subtraction with decomposition. Teach children how to exchange units of numbers. \& Children may draw dienes cubes or place value counters and cross off showing their understanding of taking away. They will need to represent any exchanging that takes place. \& Partitioning method - with exchanging Children will use the partitioning method to show exchanging. <br>
\hline \& Children continue to develop their confidence in using dienes cubes andplace value counters to show decomposition using the column method. \& Children draw representations from concrete activities using dienes cubes and place value counters. \&  <br>
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{13}{|c|}{SUBTRACTION - Upper KS2 (Years 5 \& 6)} <br>
\hline \& \multicolumn{4}{|l|}{} \& \multicolumn{6}{|l|}{Pictorial} \& \multicolumn{2}{|l|}{Abstract} <br>
\hline S
t
a
g
e

1 \& \multicolumn{4}{|l|}{Please note: Concrete apparatus and pictorial representations should still be used to support children who may be struggling with abstract concepts. Concrete apparatus should be readily available for lower achieving children and these with SEND.} \& \multicolumn{2}{|l|}{Children can draw using place value counters showing how exchanging takes place between the units of numbers.} \& \begin{tabular}{l}
15,73 <br>
0 <br>
- <br>
subtr <br>
s.

 \& 

$$
-2,582=
$$ <br>

${ }^{\text {Th }} 0000{ }^{H} 8$ <br>
btroct the 1 Is: Exch <br>
${ }^{2000} 9{ }^{\circ 00}$ <br>
the loos. 1.000 s <br>
${ }^{\text {Th }}$ <br>
tion on an

 \&  \&  \& 

Column Method <br>
Children will continue to develop their understanding of column method subtraction. <br>
Calculations will become larger.

\end{tabular} \& \[

$$
\begin{aligned}
& 5 \text { digit }-5 \text { digit } \\
& \begin{array}{rllll}
5 & 13 & 1 & & \\
6 & 7 & 6 & 9 & 7 \\
- & 6 & 8 & 5 & 4 \\
\hline 3 & 7 & 8 & 4 & 3
\end{array}
\end{aligned}
$$
\] <br>

\hline S
t
a
g
e

2 \& Introduce decimal place counters and exchange for subtracting between units numbers. \& \begin{tabular}{l}
5.74-2.2 <br>
Exchange I ten <br>
Now subtroct <br>
Now subtract

 \& 

$$
5=?
$$ <br>

for 10 hundredth <br>
e 5 hundredths. <br>
2 tenths, then the <br>

- Tih <br>
- $\varnothing \varnothing$

 \&  \& Children they have \& draw xhang \& 

d. <br>
s <br>
$\varnothing \varnothing$ <br>
$\varnothing \varnothing$
\end{tabular} \& resentati

$$
\frac{\frac{1}{100 s}}{00 ø \varnothing \varnothing}
$$

$\square$ \& ons sho

$$
\begin{aligned}
& \frac{1}{1000 s} \\
& \hline 0000 \\
& 000 \phi \sigma \\
& 0 \phi \sigma \phi \varnothing \\
& \hline
\end{aligned}
$$ \& ing where

\[
$$
\begin{array}{r}
4.264 \\
-2.437 \\
\hline 1.827
\end{array}
$$

\] \& | Children will continue to develop their understanding of column method subtraction. |
| :--- |
| Calculations will become larger, include decimal places and require 0 to be added as a placeholder. | \& | $\begin{aligned} & 6 \text { digit }-6 \text { digit } \\ & \begin{array}{rrrrrr} 5 & 13 & 1 & & & \\ 6 & 4 & 6 & 9 & 3 & 7 \\ -\begin{array}{rrrrrr} 2 & 6 & 8 & 5 & 1 & 4 \\ \hline 3 & 7 & 8 & 4 & 2 & 3 \\ \hline \end{array} \end{array} . \begin{array}{l} 4 \\ \hline \end{array} \end{aligned}$ |
| :--- |
| Numbers with 3 decimal place $\begin{array}{rrrrrr} 7 & 4 & 3 & .7 & 9 & 8 \\ - & 1 & 6 & .2 & 7 & 3 \\ \hline 5 & 2 & 7 & .5 & 2 & 5 \\ \hline \end{array}$ |
| Numbers with a different number of deamal places |
| $692-27.54$ $\begin{array}{rccc} 8 & 1 & 1 \\ 6 & 9 & 8 & 0 \\ -2 & 7 & 5 & 4 \\ 4 & 1 & 6 & 6 \end{array}$ | <br>

\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{4}{|c|}{MULTIPLICATION - KS1 (Years 1\&2)} <br>
\hline \& Concrete \& Pictorial \& Abstract <br>
\hline S
t
a
g
e

1 \& \begin{tabular}{l}
Repeated addition - Groups of multiple objects <br>
Children will count groups of the same number of objects and add them together. The children learn about grouping in practical contexts and through pictorial representations.

 \& 

Children draw representations <br>
Double 4 is 8 to show counting in multiples and groups.
\end{tabular} \& Children show multiplication as repeated addition. Children may provide pictorial representations to support. <br>

\hline S
t
a
g
e

2 \& \begin{tabular}{l}
Arrays <br>
Children will be able to represent a multiplication calculation using an array and write the multiplication symbol within a number sentence. Children will also understand that multiplication can be carried out in any order (commutative).

 \& Children draw representations to show arrays. \& 

Children use arrays to show how to solve multiplication calculations. Children are able to show that multiplication can be done in any order (commutative).

$$
\begin{aligned}
& 3 \times 5=15 \\
& 5 \times 3=15
\end{aligned}
$$ <br>

Introduce x sign and record as number sentence <br>
$7 \times 10=70$ <br>
$4 \times 5=20$ <br>
Use an array to write <br>
multiplication sentences and reinforce repeated addition. <br>
$5+5+5=15$ <br>
$3+3+3+3+3=15$ <br>
$5 \times 3=15$ <br>
$3 \times 5=15$
\end{tabular} <br>

\hline S
t
a
g
e

3 \& \begin{tabular}{l}
Number line <br>
Children will understand the operation of multiplication as repeated addition on a blank number line and will use practical resources to support this. Count the groups as children are skip counting, children may use their fingers as they are skip counting.

 \& Children will be able to use an empty number line to show multiplication as repeated addition. The use of bead string concrete resources may be used to support conceptual understanding. \& 

Children show multiplication as repeated addition.

$$
5+5+5=15
$$ <br>

Introduce x sign and record as number sentence

$$
\begin{aligned}
& 7 \times 10=70 \\
& 4 \times 5=20
\end{aligned}
$$

\end{tabular} <br>

\hline
\end{tabular}

| MULTIPLICATION - Lower KS2 (Years 3 \& 4) |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Concrete | Pictorial | Abstract |
| S t a g e 1 1 | Number line - Consolidation <br> Children will understand the operation of multiplication as repeated addition on a blank number line and will use practical resources to support this. Count the groups as children are skip counting, children may use their fingers asthey are skip counting. | Children will be able to use an empty number line to show multiplication as repeated addition. The use of bead string concrete resources may be used to support conceptual understanding. | Children show multiplication as repeated addition. $5+5+5=15$ <br> Introduce x sign and record as number sentence $\begin{aligned} & 7 \times 10=70 \\ & 4 \times 5=20 \end{aligned}$ |
|  | Partitioning <br> Children will learn to multiply ones and tens separately before recombing the numbers back together. They can use Dienes cube of place value counters to achieve this. | Children can draw representations of the partitioning process to support their conceptual understanding. | Partition a number and then multiply each part before recombining it back together. $\begin{aligned} & 27 \times 5= \\ & 20 \times 5=100 \\ & 7 \times 5=\frac{35}{135} \end{aligned}$ |
|  | Grid Method <br> Show the links with arrays to first introduce the grid method. <br> Move onto Dienes cubes to move towards a more compact method. <br> Move on to place value counters to show how we are finding groups of a number. We are multiplying by 5 so weneed 5 rows of that number.  | Pictorial representations can be made using their concrete manipulatives as visuals. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their calculations. |  |

\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{4}{|c|}{MULTIPLICATION - Upper KS2 (Years 5 \& 6)} <br>
\hline \& Concrete \& Pictorial \& Abstract <br>
\hline S
t
a
g
e

1 \& \begin{tabular}{l}
Concrete materials may be needed conceptual understanding. Dienes counters will support. <br>
When multiplying by 10,100,1000 initial concrete resources willbe used to support understanding.

 \& 

Use place value equipment to compare methods. <br>
Method I <br>
Method 2

 \& 

The grid method will lead onto expanded method and then onto the compact short multiplication. <br>
Use known facts and unitising to multiply.

$$
\begin{aligned}
& 5 \times 4=20 \\
& 5 \times 40=200 \\
& 5 \times 400= \\
& 2,000
\end{aligned}
$$

\end{tabular} <br>

\hline S
t
a
g
e

2 \& When multiplying decimals by 10,100,1000 initial concrete resources will be used to supportunderstanding to show how exchanging can take place. \& This pictorial grid method will support children's understanding of multiplying by $10,100,1000$. \& \begin{tabular}{lrl}
Long multiplication <br>
Children may wish to use <br>
2 separate calculations to \& 23 \& <br>

| support their |
| :--- | \& $\frac{13}{+69}$ \& $(3 \times 23)$ <br>


| understanding. Reinforce |
| :--- |
| language of place value | \& 230 \& $(10 \times 23)$ <br>


| when multiplying by |
| :--- |
| multiples of 10. Extend to 3 |
| multiplied by a 2-digit number. | \& \& <br>

\&

 

4-digit numbers
\end{tabular} <br>

\hline S
t
a
g
e

3 \& \begin{tabular}{l}
Please note: <br>
Concrete apparatus and pictorial struggling with abstract concepts children and these with SEND.

 \& still be used to support children who maybe ould be readily available for lower achieving \& 

Use column multiplication, ensuring understanding of place value at each

$$
\begin{array}{r}
1 . \\
\times \quad 3 \\
\times \\
\hline 8.5 \\
\hline 2
\end{array}
$$ <br>

stage.
\end{tabular} <br>

\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{4}{|c|}{DIVISION - KS1 (Years 1\&2)} <br>
\hline \& Concrete \& Pictorial \& Abstract <br>
\hline S
t
a
g
e

1 \& \begin{tabular}{l}
Sharing and Grouping Divide quantities into equal groups. Use cubes, counters,objects or place value counters to aid understanding. <br>
How many $2 s$ in 10 ?

 \& 

Use pictures to share objects. Use circles rather than dots to aid counting. <br>
Share 10 into 2 equal groups <br>
How many $2 s$ in 10? <br>
Develop division as repeated subtraction on a number line.

 \& 

Children will be able to represent a division calculation using a pictorial method and write the division within a number sentence.

$$
10 \div 2=5
$$ <br>

Share 10 into 2 equal groups
\end{tabular} <br>

\hline S
t
a
g
e

2 \& | Arrays |
| :--- |
| Link division to multiplication by creating an array and thinking aboutthe number sentences that can be created. $\begin{array}{ll} \text { Eg: } \\ 15 \div 3=5 & 5 \times 3=15 \\ 15 \div 5=3 & 3 \times 5=15 \end{array}$ | \& Draw arrays to show how pictures are divided. \& Children will be able to represent a division calculation using an array and write the division within a number sentence <br>

\hline S
t
a
g
e

3 \& | Repeated addition and subtraction |
| :--- |
| Children will understand the operation and repeated addition or subtraction using bead strings and numberlines. This will support the $15 \div 3=5$ pictorialelement. | \& Children will understand the operation of division as grouping using repeated addition or subtraction on a prepared number line.

$$
12 \div 3=4
$$ \& Children will be able to represent a division calculation using a numberline and write the division within a number sentence.

$$
12 \div 3=4
$$ <br>

\hline
\end{tabular}

|  | DIVISION - Lower KS2 (Years 3 \& 4) |  |  |
| :---: | :---: | :---: | :---: |
|  | Concrete | Pictorial | Abstract |
| S | Division with no remainders through sharing.Use concrete materials to share into groups. | Consolidate learning from KS1 using diagrams of sharing and repeated subtraction and addition on a number line to make jumps <br> Example without remainder: $40 \div 5$ <br> Ask "How many 5s in 40?" <br> Concrete methods could be represented pictorially within books to show understanding. | How many groups of 6 in 24 ? $24 \div 6=4$ <br> Abstract methods may be supported with pictorial methods within the children's books. |
|  | Division with remainder through sharing $14 \div 3=$ <br> Divide objects between groupsand see how much is left over. <br> Division no remainders - introduction to bus stop method Use place value equipment on a placevalue grid alongside short division. The model uses grouping. | Students can continue to use drawn diagrams with circles to help them divide numbers into equal groups. <br> Remainders will be seen by not fitting into a whole group. <br> Draw dots and group them to divide an amount and clearly show a remainder. <br> Example without remainder. $40 \div 5$ <br> Ask "How many 5 s in 40? <br> Example with remainder: <br> $38 \div 6$ | Children will begin to move onto division with remainders. A number sentence will support any abstract written calculation by using pictorial method to support. <br> Short division <br> Children will begin to use the formal written method of division without remainders. This will only come after a clear concept is understood using manipulatives. |




## Dividing decimals by 10,100 and 1,000

Use place value counters to represent dividing by 10, 100,
1000. Represent division using exchange on a place valuegrid.


Eschange each of for ten 001 s


Divide 20 counters by 10 .
$0 \cdot 2$ is 2 tenths.
2 tenths are equivalent to 20 hundredths. 20 hundredths divided by 10 is 2 hundredths.

Represent division to show the relationship with multiplication. Understand the effect of dividing by 10, 100 and 1,000 on the digits on a place value grid.

| 0 | $\bullet$ | Tth | Hth | Thth |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\bullet$ | 8 | 5 |  |
| 0 | $\bullet$ | $>$ | $\mathbf{S}_{8}$ | $>5$ |

$$
0.85 \div 10=0.085
$$


$8.5 \div 100=0.085$

Finally move into decimal places to divide the total accurately using a formal method for division (Bus stop)


## Long Division

Calculations will start with tens and ones and move onto more advanced division calculations.


With questions that end with a remainder, children will be taught to interpret this as either a fraction orrounded $372 \div 15=24 \frac{4}{5}$ according to the context of the question.
$-\quad 60$

